



Pendle
Education Trust



Calculation Policy

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Mathematical Terminology

This calculation policy uses mathematical terminology that may be new or unfamiliar. They are terms that your child should be familiar with, depending on his or her Year Group. Also included are some standard terms used in class.

Addend- Any of the numbers that are added together. E.g. In $8 + 3 = 11$, the 8 and the 3 are addends.

Array- Items (such as objects, numbers, etc.) arranged in rows and/or columns.

Bridging through ten- When performing additions or subtractions, counting through a tens boundary is called 'bridging through ten'. It helps to count up to the boundary first, then through it. E.g. $18 + 4$ will bridge through the boundary from 19 to 20, so we would count $18 + 2$, then $20 + 2$.

Column Addition- Addition by writing one number below the other and then adding one column at a time.

Counting back- As with counting on, but in subtraction.

Counting on- When performing a calculation, such as $8 + 3$, start at 8 and count: 9, 10, 11. You can count on or back in tens or hundreds etc. as well as ones.

Commutative Law- The law that states that: a multiplication or addition can be made in any order to arrive at the same product or sum respectively. E.g. $8 + 5 + 2 = 15$; $2 + 5 + 8 = 15$ and $5 + 8 + 2 = 15$. These calculations are said to demonstrate commutativity.

Decrease- Making something smaller (in size or quantity).

Difference- The result of a subtraction calculation.

Dividend- A number that is to be divided in a calculation.

Divisor- The number that the dividend is to be divided by.

Empty Number-Lines- A number-line without numbers. They are used when calculations involving numbers greater than 10. It would be impractical to calculate $87 - 23$ with a printed number-line starting at 0. A line is drawn and the only numbers written on the line are the starting number (which depends on the method used) and the numbers arrived at whilst 'jumping' along it.

Estimate- To find a value that is close enough to the right answer, usually with some thought or calculation involved.

Exchanging- In multiplication, the act of giving or taking one thing in exchange for another e.g. tens.

Factors- Factors are whole numbers that can be multiplied to make a given whole number. E.g. 4 and 3 are factors of 12, so are 6 and 2.

Grid Method- A method for introducing written multiplications where the numbers involved are partitioned before multiplying. The products are recombined as they will obey Commutative Law.

Increase- Making something bigger (in size or quantity).

Inverse Operations- The opposite of an operation. Inverse operations are used to find a missing value in a number sentence. The inverse of an addition is a subtraction and vice-versa. The inverse of a multiplication is a divide. However, the inverse of a divide calculation is not always a multiplication:

$$5 + 15 = 20 \rightarrow 20 - 5 = 15 \text{ (and vice-versa)}$$

$$3 \times 4 = 12 \rightarrow 12 \div 4 = 3 \text{ BUT...}$$

$$12 \div 4 = 3 \rightarrow 3 \times 12 \text{ is not } 4! \text{ (You have to do } 12 \div 3 \text{ to get } 4).$$

Manipulative- any concrete objects that allow students to explore an idea in a hands-on approach.

Minuend- The first number in a subtraction. The number from which another number (the subtrahend) is to be subtracted. E.g. $8 - 3 = 5$

Multiple- The result of multiplying a number by an integer (not by a fraction).

Multiplicand- The number that gets multiplied. E.g. $6 \times 3 = 18$

Number Sentence- An equation written in maths. Like in literacy, they must make sense. = means 'the same as'; > means 'more than' and < means 'less than'.

E.g. $6 + 8 = 20$ WRONG

$$6 + 8 > 20 \text{ WRONG}$$

$$6 + 8 < 20 \text{ CORRECT because it makes sense.}$$

Number Bonds- Number 'facts' that children will learn in KS1 and continue using to aid mental calculations. Number bonds to ten are the following: $1 + 9$; $2 + 8$; $3 + 7$; $4 + 6$ and $5 + 5$. There are also number bonds to 20, 100, 1000 and so on.

Operation- A procedure that produces a new value from two or more input values. + - x and \div are the four main arithmetic operations.

Partitioning- Splitting a number greater than 10 into its constituent thousands, hundreds, tens and ones. E.g. $4723 \rightarrow 4000$ and 700 and 20 and 3 .

Place Value- Place value determines the value of a digit depending on where it sits in the written number. E.g. $4562 \rightarrow$ The '5' represents 500 in this case.

Prime Number- A number with only one factor. Every whole number greater than one is a product of prime numbers.

Product- The result of a multiplication calculation.

Quotient- The result of a division calculation. It is arrived at by counting groups of the divisor up to the total of the dividend.

Recombining- After partitioning and calculating, putting the numbers 'back together'.

Remainder- When dividing, the remainder is how much of the divisor is needed to reach the dividend total when the quotient is not a whole number.

Subtrahend- The number that is to be subtracted. The second number in a subtraction. E.g. $8 - 3 = 5$

Sum- The result of an addition calculation.

Calculation Policy Guidance

This policy supports both the Red Rose Maths Mastery and the White Rose Maths schemes used throughout the school. Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum. This calculation policy should be used to support children to develop a deep understanding of number and calculation. This policy has been designed to teach children through the use of concrete, pictorial and abstract representations.

- **Concrete representation**— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a ‘hands on’ component using real objects and is a foundation for conceptual understanding.
- **Pictorial representation** – a pupil has sufficiently understood the ‘hands on’ experiences performed and can now relate them to representations, such as a diagram or picture of the problem.
- **Abstract representation**—a pupil is now capable of representing problems by using mathematical notation, for example $12 \times 2 = 24$.

It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations.

Mathematics Mastery

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

How to use the policy:

This mathematics policy is a guide for all staff in St James’ and has been adapted from work by the NCETM. Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [make it, draw it, write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group’s scheme of work.

Addition: Mental Calculations

These are a selection of mental calculation strategies:

Mental recall of number bonds

$$\begin{array}{ll} 6 + 4 = 10 & ___ + 3 = 10 \\ 25 + 75 = 100 & 19 + ___ = 20 \end{array}$$

Use near doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$



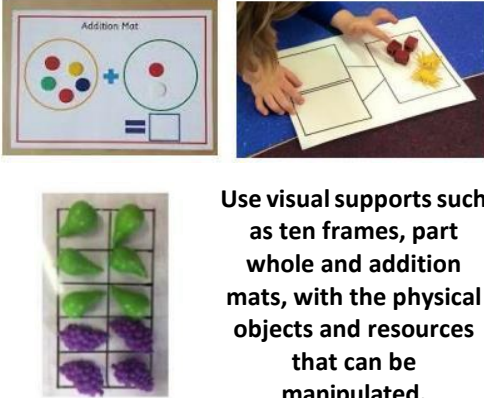
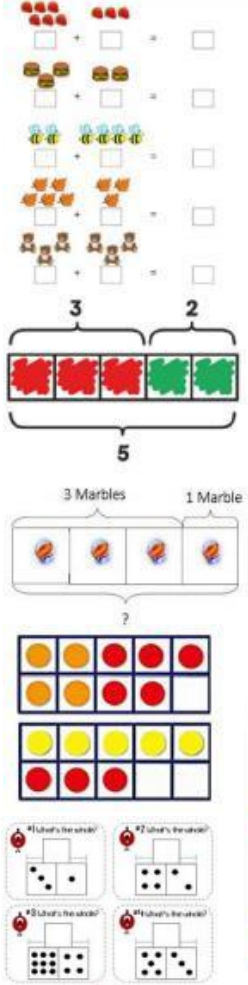
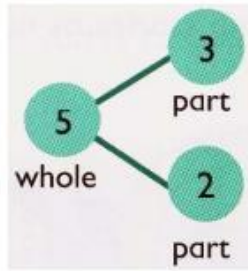
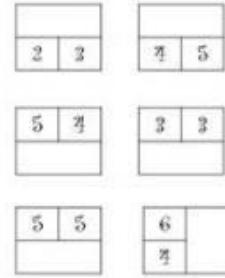
Use the relationship between addition and subtraction

$$36 + 19 = 55 \quad 55 - 19 = 36$$

$$19 + 36 = 55 \quad 55 - 36 = 19$$

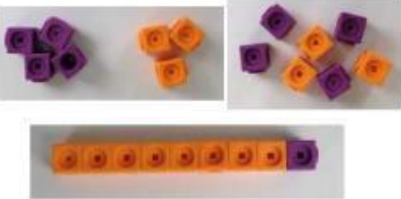
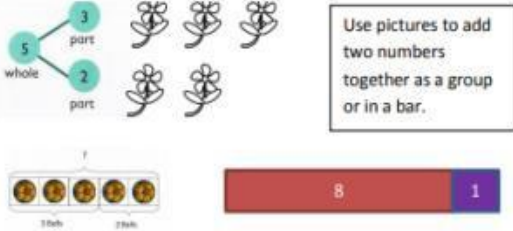


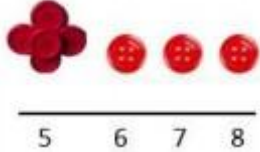

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

Addition: Written Calculations

Addition- EYFS			
Objectives	Concrete	Pictorial	Abstract
<ul style="list-style-type: none"> - Know that a group of things change in quantity when something is added. - Find the total number of items in two groups by counting all of them. - Say the number that is one more than a given number - Find one more from a group of up to five objects, then ten objects - In practical activities and discussion, beginning to use the vocabulary involved in adding - Using quantities and objects, add two single digit numbers and count on to find the answer - Solve problems including doubling 	 <p>Use toys and general classroom resources for children to physically manipulate, group/regroup.</p>  <p>Use specific maths resources such as counters, snap cubes, Numicon etc.</p>  <p>Use visual supports such as ten frames, part whole and addition mats, with the physical objects and resources that can be manipulated.</p>	 <p>Two groups of pictures so children are able to count.</p> <p>Bar model using visuals, pictures/icons or colours.</p> <p>Use visual supports such as ten frames, part whole and addition mats with pictures/icons.</p>	<p>A focus on symbols and numbers to form a calculation.</p> $5 + 2 = 7$   <p>Note- No expectation for children to be able to record a number sentence/addition calculation.</p>

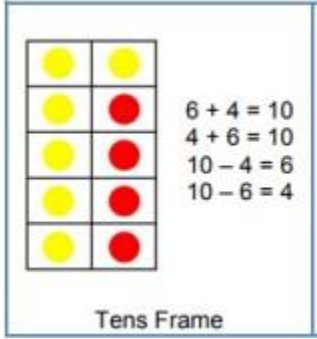
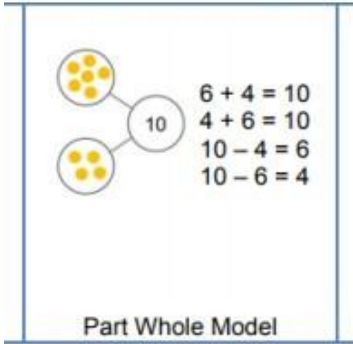
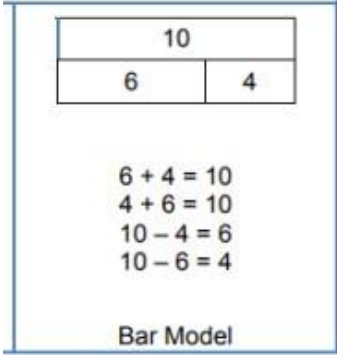

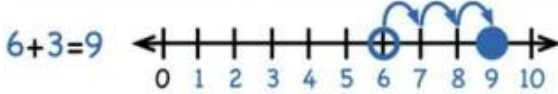



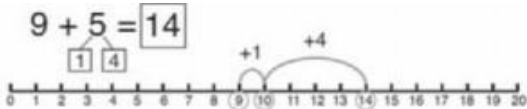
Addition- Year 1

Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, whole

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Number bonds of 5, 6, 7, 8, 9 and 10</p> <p>Combining two parts to make a whole: part-whole model</p>	 <div style="display: flex; align-items: center; margin-top: 10px;"> <div style="border: 1px solid gray; padding: 5px; margin-right: 10px;"> <p>10</p> </div> <div style="margin-right: 10px;"> <p>5</p> </div> <div style="margin-right: 10px;"> <p>5</p> </div> </div> <p>Use cubes to add numbers together as a group or in a bar. (Some children still need to use real objects)</p> <p>Use the part-whole model.</p>	 <div style="border: 1px solid gray; padding: 5px; margin: 10px 0; font-size: small;"> Use pictures to add two numbers together as a group or in a bar. </div> <p>The Bar Model will be continued from EYFS as a method to support problem solving involving addition, continuing with the concrete representations and moving onto using pictorial representations of objects. Some children will also move onto the abstract.</p>	 <p>Using the part-whole model (left) to move into the abstract.</p> <p>$4 + 3 + 7$</p> <p>$10 = 6 + 4$</p>
<p>Counting</p> <p>Start with a larger number and count on using a bead string/number line</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p> 	<p>Use a number line to count on in ones.</p> 	<p>$5 + 3 = 8$</p>


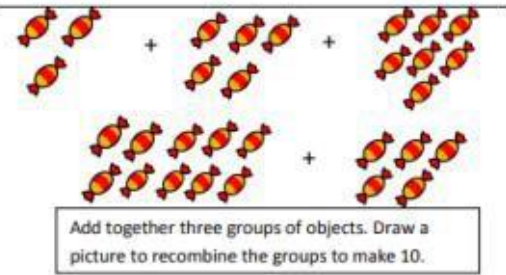
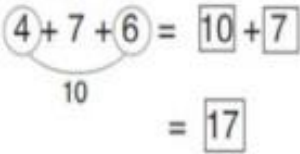
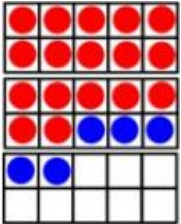
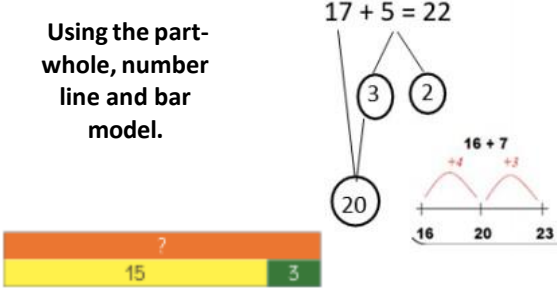

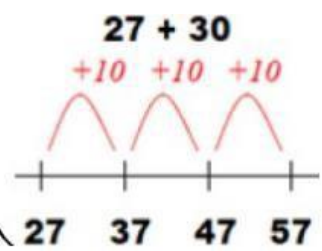
Addition- Year 1

Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, whole

Objective and Strategy	Concrete	Pictorial	Abstract
<p style="color: blue;">Represent and use number bonds and related subtraction facts within 20</p>	 <p style="text-align: center;">Tens Frame</p> <p>(Some children may need to initially use real objects then move onto the representation, egg boxes may also be used to support this)</p>	 <p style="text-align: center;">Part Whole Model</p>	 <p style="text-align: center;">Bar Model</p> <p>Bar model and part-whole to be used alongside abstract.</p>
<p style="color: blue;">Addition and subtraction of one-digit and two-digit numbers to 20 including 0</p>		<p>$6 + 3 = 9$</p>  <p>Start at the larger number on the number line and count on in ones.</p>	<p>$5 + 12 = 17$ $17 = 12 + 5$</p>
<p style="color: blue;">Regrouping to make 10. The 'Make 10' strategy</p>	  <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p>	 <p>$3 + 9 =$</p>  <p>Use pictures or a number line. Regroup or partition the smaller number using the part-whole model to make 10.</p>	<p style="text-align: center;">$7 + 4 = 11$</p> <p style="text-align: center;">If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>

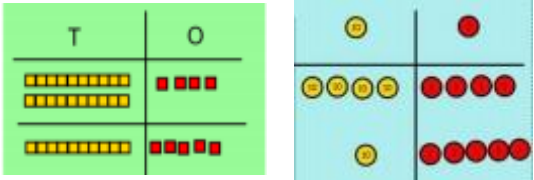
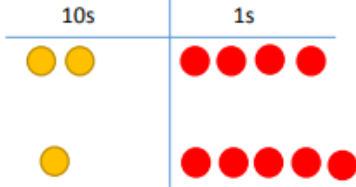
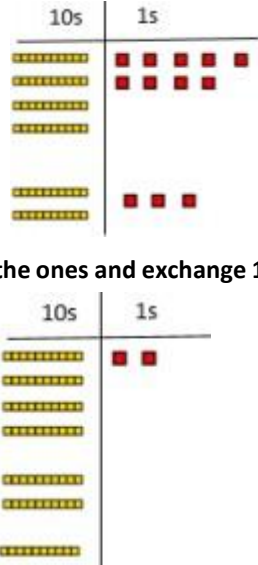
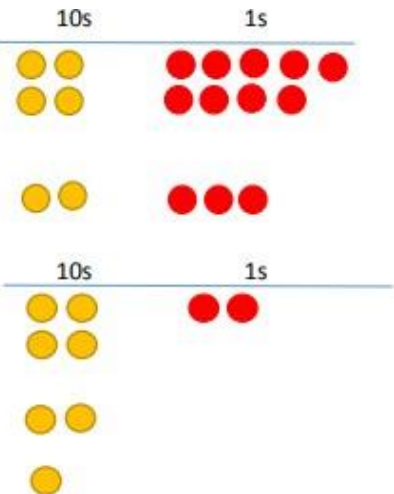
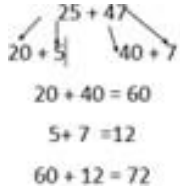
Addition- Year 2

Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Objective and Strategy	Concrete	Pictorial	Abstract				
Adding 3 1-digit numbers	<p>4 + 7 + 6 = 17 Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p>		 <p>Combine the two numbers that make 10 and then add on the remainder.</p>				
Adding a 2-digit number and ones	 <p style="text-align: center;">17 + 5 = 22</p> <p style="text-align: center;">Use ten frame to make 'magic ten'</p> <p style="text-align: center;">Children explore the pattern: 17 + 5 = 22 27 + 5 = 32</p>	<p>Using the part-whole, number line and bar model.</p> <p>17 + 5 = 22</p> 	<p>17 + 5 = 22</p> <p>Explore related facts</p> <p>17 + 5 = 22 5 + 17 = 22 22 - 17 = 5 22 - 5 = 17</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="2" style="text-align: center;">22</td></tr> <tr><td style="width: 50%;">17</td><td style="width: 50%;">5</td></tr> </table>	22		17	5
22							
17	5						
Adding a 2-digit number and multiples of 10	 <p style="text-align: center;">25 + 10 = 35</p> <p>Explore that the ones digit does not change.</p>	<p style="text-align: center;">27 + 30</p> <p style="text-align: center;">+10 +10 +10</p>  <p>Base 10 may be used above the number line initially.</p> <p>The calculation will be shown alongside the number line to see the connection.</p>	<p>27 + 10 = 37 27 + 20 = 47 27 + □ = 57</p>				

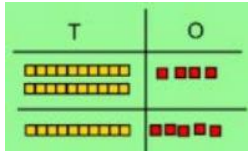
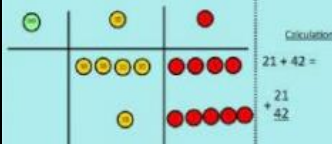
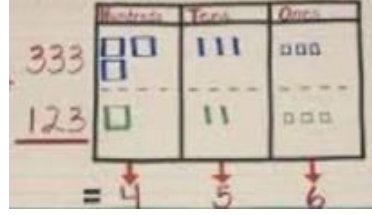
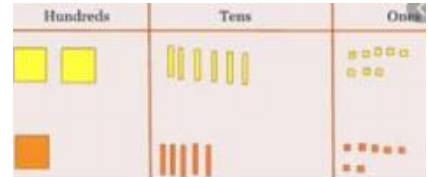
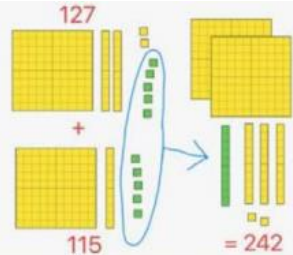
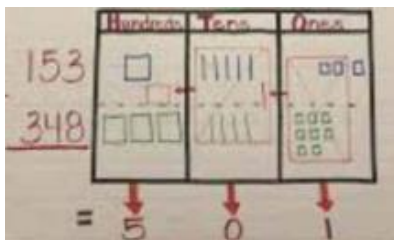
Addition- Year 2

Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Objective and Strategy	Concrete	Pictorial	Abstract
<p style="color: #0070c0; font-weight: bold;">Column method without regrouping</p>	<p>Add together the ones first, then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p> <p>24 + 15 = 44 + 15 =</p> 	<p>After physically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p> 	<p>24 + 15 = 39</p> $\begin{array}{r} 24 \\ + 15 \\ \hline 39 \end{array}$
<p style="color: #0070c0; font-weight: bold;">Column method with regrouping</p>	<p>Make both numbers on a place value grid.</p>  <p>Add up the ones and exchange 10 ones for 1 ten.</p>	<p>Using place value counters, children can draw the counters to help them to solve additions.</p> 	<p style="text-align: center;">Partitioning:</p>  <p>Recording addition in columns supports place value and prepares for formal written methods with larger numbers. Toward the end of the year, children move to more formal recording using partitioning method:</p> $\begin{array}{r} 40 + 7 \\ 30 + 5 \\ \hline 70 + 12 \end{array}$


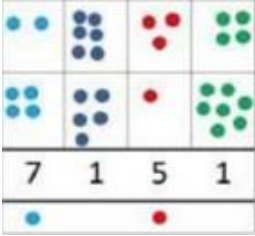
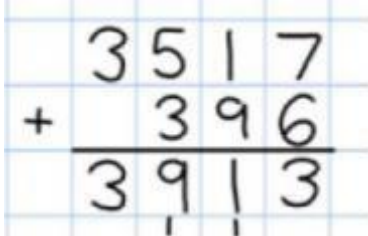

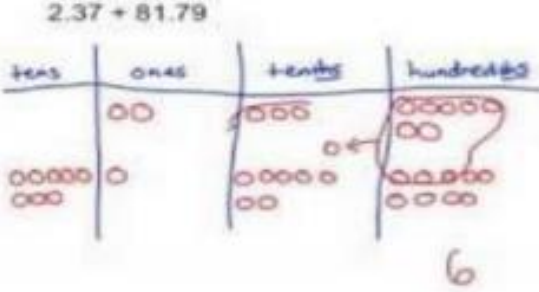
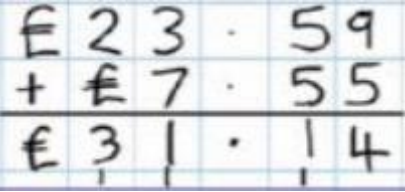
Addition- Year 3

Vocabulary: addition, add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary

Objective and Strategy	Concrete	Pictorial	Abstract
<p style="color: #0070c0;">Add and subtract numbers with up to 3-digits, using formal written methods of columnar addition</p> <p style="color: #0070c0;">Column addition (no regrouping)</p>	<div style="text-align: center;">  </div> <p>Using manipulatives (dienes, numicon, counters), children are to line up hundreds, tens and ones.</p> <div style="text-align: center;">  </div> <p>Children should be secure with using PV counters before moving onto pictorial.</p>	<div style="text-align: center;">  </div> <p>Children are to draw, in a PV frame, the manipulatives, that they are using.</p> <p>Secure knowledge of representation with the PV columns.</p>	<p style="text-align: center;">Abstract</p> $\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Children to move onto recording more formally. Some children may need to use the expanded method (see below).</p>
<p style="color: #0070c0;">Column addition (with regrouping)</p>	<div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <p>Exchange ten ones for a ten. Model using dienes, numicon and place value counters.</p>	<div style="text-align: center;">  </div> <p>Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line.</p>	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ \hline 60 + 13 = 73 \end{array}$ <p>Children are to begin with the abstract: expanded form. For those children that are confident after AFL, the below method should be used.</p>

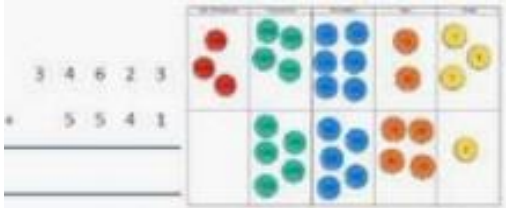
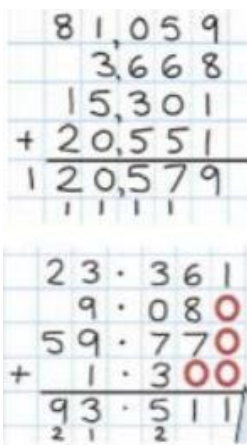
Addition- Year 4

Vocabulary: addition, add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point

Objective and Strategy	Concrete	Pictorial	Abstract
<p style="color: #0070c0;">Using formal written methods of columnar addition where appropriate</p> <p style="color: #0070c0;">Add numbers with up to 4 digits (with exchange)</p>	<p>Children continue to use dienes or place value counters to add, exchanging ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p>	 <p>Continue from previous work to carry hundreds as well as tens.</p>
<p style="color: #0070c0;">Add decimals with 2 decimal places, including money</p>	 <p>Introduce decimal place value counters and model exchange for addition.</p>		 <p>As the children move on, introduce decimals with the same number of decimal places and different money can be used here.</p>

Addition- Years 5 and 6

Vocabulary: addition, add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point

Objective and Strategy	Concrete	Pictorial	Abstract
<p style="color: #0070c0; font-weight: bold;">Add numbers with more than 4 digits</p>	See Year 4	See Year 4	<div style="text-align: center;">  </div> <p style="text-align: center; font-weight: bold; margin-top: 10px;">Children should have abstract supported by a pictorial or concrete if needed.</p>
<p style="color: #0070c0; font-weight: bold;">Add decimals with 2 decimals places, including money</p>	See Year 4	See Year 4	<div style="text-align: center;">  </div> <p style="text-align: center; font-weight: bold; margin-top: 10px;">Insert zeros for place holders.</p>

Subtraction: Mental Calculations

These are a selection of mental calculation strategies:

Mental recall of addition and subtraction facts

$$10 - 6 = 4 \quad 17 - \underline{\quad} = 11$$

$$20 - 17 = 3 \quad 10 - \underline{\quad} = 2$$

Find a small difference by counting up

$$82 - 79 = 3$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$




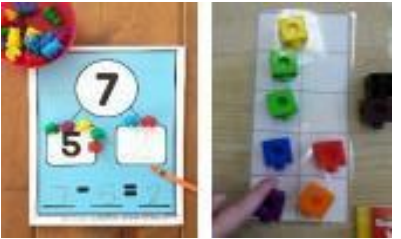
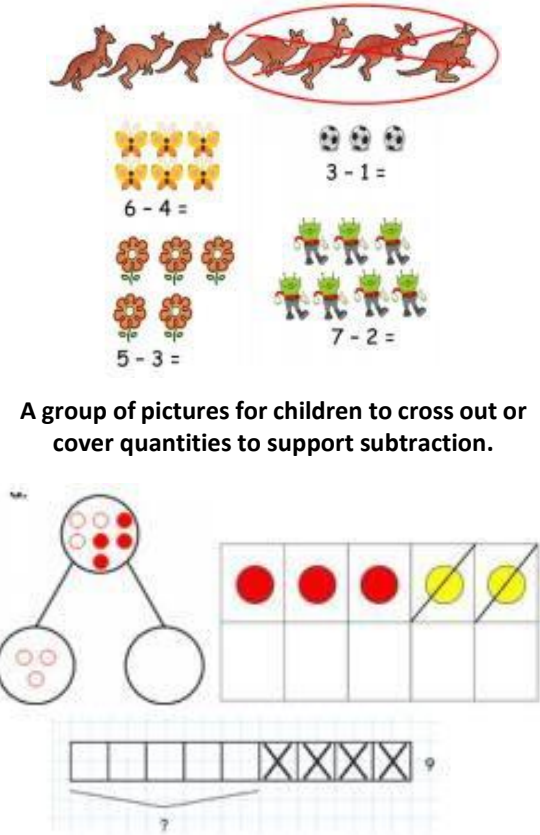

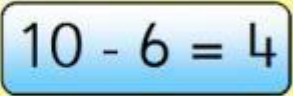
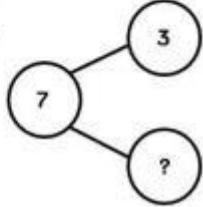
Use the relationship between addition and subtraction

$$36 + 19 = 55 \quad 55 - 19 = 36$$

$$19 + 36 = 55 \quad 55 - 36 = 19$$

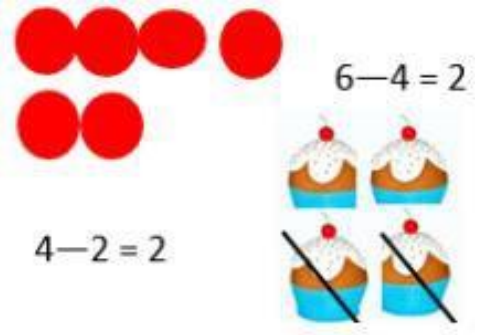
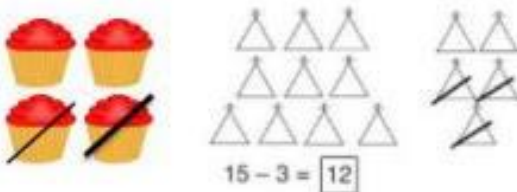
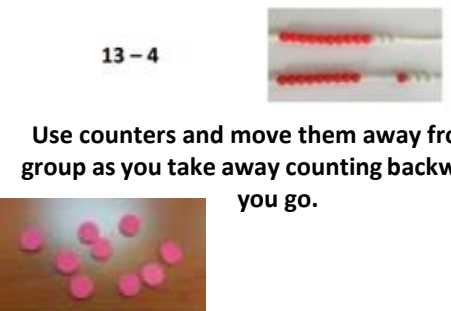
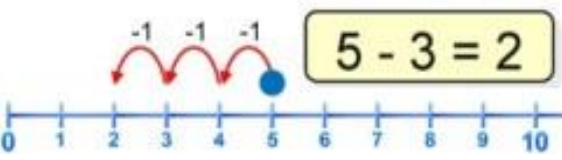
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

Subtraction: Written Calculations

Subtraction- EYFS							
Objectives	Concrete	Pictorial	Abstract				
<ul style="list-style-type: none"> - Know that a group of things change in quantity when something is taken away. - Find one less from a group of five objects, then ten objects. - In practical activities and discussion, begin to use the vocabulary involved in subtracting. - Using quantities and objects, subtract two single digit numbers and count back to find the answer. 	 <p>Use toys and general classroom resources for children to physically manipulate, group/regroup.</p>   <p>Use specific maths resources such as bead strings, snap cubes, Numicon etc.</p>  <p>Use visual supports such as ten frames, part whole and addition mats, with the physical objects and resources that can be manipulated.</p>	 <p>A group of pictures for children to cross out or cover quantities to support subtraction.</p> <p>Use visual supports such as ten frames, part whole and addition mats with pictures/icons.</p>	<p>A focus on symbols and numbers to form a calculation.</p>   <table border="1" data-bbox="1653 842 1955 962"> <tr> <td>3</td> <td>?</td> </tr> <tr> <td colspan="2">7</td> </tr> </table> <p>$7 - 3 = ?$</p>  <p>Note- No expectation for children to be able to record a number sentence/subtraction calculation.</p>	3	?	7	
3	?						
7							

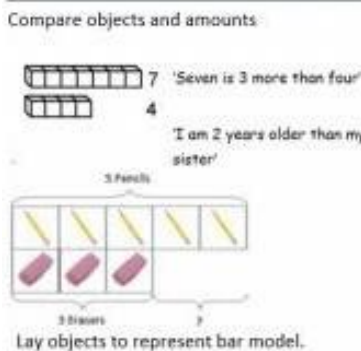
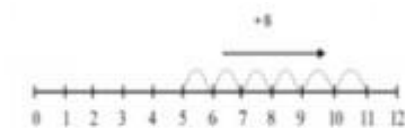
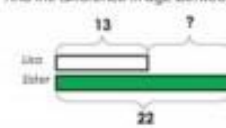
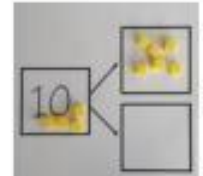
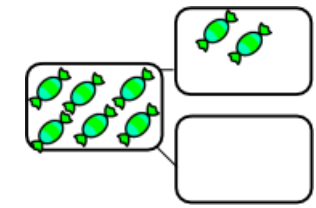
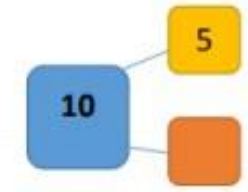
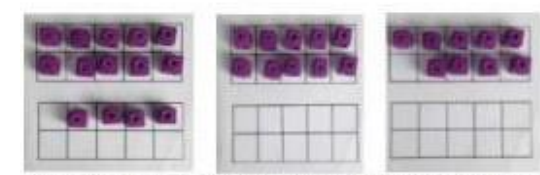
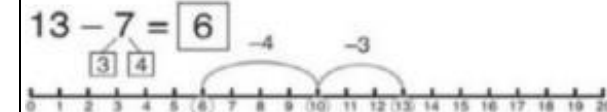
Subtraction- Year 1

Vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Subtract one-digit and two-digit numbers to 20, including 0.</p> <p>Taking away ones.</p>	<p>Use physical objects, counters, cubes etc. to show how objects can be taken away.</p>  <p>$6 - 4 = 2$</p> <p>$4 - 2 = 2$</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p>	<p>$7 - 4 = 3$</p> <p>$16 - 9 = 7$</p>
<p>Counting back</p> <p>Start with a larger number and count back using a bead string/number line.</p>	<p>Make the larger number in your subtraction. Move the beads along as your bead string as you count backwards in ones.</p> <p>$13 - 4$</p>  <p>Use counters and move them away from the group as you take away counting backwards as you go.</p>	 <p>Count back on a number line or track. Start at the bigger number and count back to the smaller number showing the jumps on the number line.</p>	<p>Put 13 in your head, count back 4. What number are you at? (Use your fingers to help you).</p>

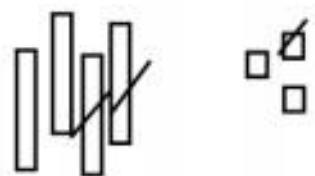
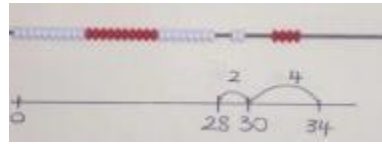
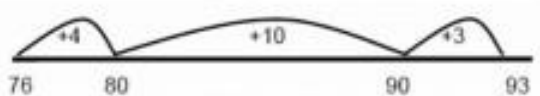
Subtraction- Year 1

Vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is

Objective and Strategy	Concrete	Pictorial	Abstract
Find the difference	<p>Compare objects and amounts:</p>  <p>7 'Seven is 3 more than four' 4 'I am 2 years older than my sister'</p> <p>3 pencils 3 blades 7</p> <p>Lay objects to represent bar model.</p>	 <p>Count on to find the difference.</p> <p>Comparison Bar Models</p> <p>Draw bars to find the difference between 2 numbers.</p> <p>Lia is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p> 	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?</p>
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Part- whole model</p>	 <p>10 – 6 = 4</p> <p>Link to addition. Use part-whole model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p>	 <p>Use a pictorial representation of objects to show the part-whole model.</p>	 <p>Move to using numbers within the part whole model.</p>
Make 10	<p>14 – 9 =</p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.</p>	<p>13 – 7 = 6</p>  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p>16 – 8 =</p> <p>How many do we take off to reach the next 10? How many do we have left to take off?</p>

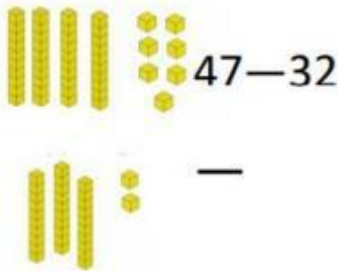
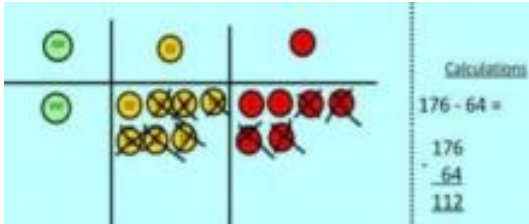

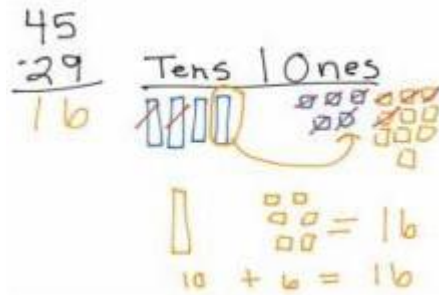
Subtraction- Year 2

Vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Objective and Strategy	Concrete	Pictorial	Abstract				
<p style="color: #0070c0;">Subtract a two-digit number and ones, a two-digit number and tens, two two-digit numbers.</p> <p style="color: #0070c0; text-align: center;">Partitioning to subtract without regrouping: 'Friendly numbers'</p>	<p style="text-align: center;">$34 - 13 = 21$</p> <p>Use dienes to show how to partition the number when subtracting without regrouping.</p> <p>This calculation will be shown alongside the manipulative used.</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 2px;">Model</th> <th style="padding: 2px;">Calculation</th> </tr> </thead> <tbody> <tr> <td style="height: 20px;"></td> <td style="height: 20px;"></td> </tr> </tbody> </table>	Model	Calculation			<p>Children draw representations of Dienes and cross off.</p> <div style="text-align: center;">  <p style="margin-top: 10px;">$43 - 21 = 22$</p> </div>	<p style="font-size: 1.2em; font-weight: bold; color: #0070c0;">$43 - 21 = 22$</p> <p><i>Recording subtraction in columns supports place value and prepares for formal written methods with larger numbers. Toward the end of the year, children move to more formal recording using partitioning method:</i></p> <p>e.g. $43 - 21 = 22$</p> <div style="margin-left: 20px;"> <p>40 and 3</p> <p><u>-20 and 1</u></p> <p>20 and 2</p> </div>
Model	Calculation						
<p style="color: #0070c0;">Make 10 strategy</p>	<div style="text-align: center;">  </div> <p style="text-align: center; font-weight: bold;">$34 - 28$</p> <p>Use a bead bar or bead strings to model counting to next ten and the rest</p>	<div style="text-align: center;">  <p style="margin-top: 5px;">'counting on' to find 'difference'</p> </div> <p>Use a number line to count on to next ten and then the rest.</p>	<p style="font-size: 1.2em; font-weight: bold; color: #0070c0;">$93 - 76 = 17$</p>				

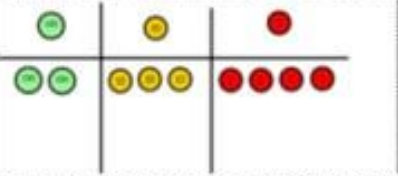
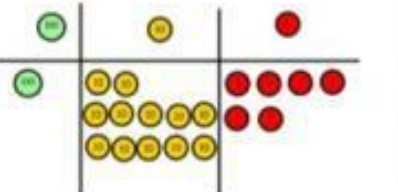
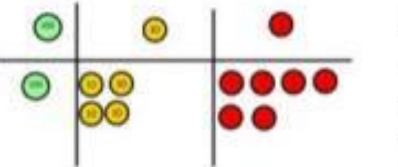
Subtraction- Year 3

Vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Objective and Strategy	Concrete	Pictorial	Abstract																														
<p>Subtract numbers with up to three-digits, using formal written methods of columnar subtraction</p> <p>Column subtraction (without exchanging)</p>	 <p style="text-align: right; font-size: 2em;">$47 - 32$</p> <p style="text-align: center;">Use base 10 or Numicon to model</p> <p style="text-align: center;">This calculation will be shown alongside the model chosen to see the calculation.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 5px;">Model</th> <th style="padding: 5px;">Calculation</th> </tr> </thead> <tbody> <tr> <td style="height: 30px;"></td> <td></td> </tr> </tbody> </table>	Model	Calculation			<p>Children are to be secure with use of PV counters before moving onto abstract.</p> 	<p>Children should begin with the expanded form. Moving onto a more formal way as below.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ </div> <div style="border: 1px solid black; padding: 5px;"> $728 - 582 = 146$ <table style="font-size: 0.8em; border-collapse: collapse;"> <tr><td style="text-align: center;">h</td><td style="text-align: center;">t</td><td style="text-align: center;">u</td></tr> <tr><td style="text-align: center;">7</td><td style="text-align: center;">2</td><td style="text-align: center;">8</td></tr> <tr><td style="text-align: center;">5</td><td style="text-align: center;">8</td><td style="text-align: center;">2</td></tr> <tr><td colspan="3" style="border-top: 1px solid black;"></td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">4</td><td style="text-align: center;">6</td></tr> </table> </div> </div>	h	t	u	7	2	8	5	8	2				1	4	6											
Model	Calculation																																
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<p>Subtract numbers with up to three-digits, using formal written methods of columnar subtraction</p> <p>Column subtraction (with exchanging)</p>	<p>Begin with base 10 or Numicon. Move to place value counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>  <p style="font-size: 0.8em;">Column method (using base 10 and having to exchange)</p> <p style="font-size: 0.8em;">$45 - 26$</p> <ol style="list-style-type: none"> 1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens. 	 <p style="text-align: center;">When confident, children can find their own way to record the exchange/regrouping.</p>	<p>Children should begin with the expanded form. Moving onto a more formal way as below (bottom picture).</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px;"> $836 - 254 = 582$ <table style="font-size: 0.8em; border-collapse: collapse;"> <tr><td style="text-align: center;">h</td><td style="text-align: center;">t</td><td style="text-align: center;">u</td></tr> <tr><td style="text-align: center;">800</td><td style="text-align: center;">30</td><td style="text-align: center;">6</td></tr> <tr><td style="text-align: center;">200</td><td style="text-align: center;">50</td><td style="text-align: center;">4</td></tr> <tr><td colspan="3" style="border-top: 1px solid black;"></td></tr> <tr><td style="text-align: center;">500</td><td style="text-align: center;">80</td><td style="text-align: center;">2</td></tr> </table> </div> <div style="border: 1px solid black; padding: 5px;"> $728 - 582 = 146$ <table style="font-size: 0.8em; border-collapse: collapse;"> <tr><td style="text-align: center;">h</td><td style="text-align: center;">t</td><td style="text-align: center;">u</td></tr> <tr><td style="text-align: center;">7</td><td style="text-align: center;">2</td><td style="text-align: center;">8</td></tr> <tr><td style="text-align: center;">5</td><td style="text-align: center;">8</td><td style="text-align: center;">2</td></tr> <tr><td colspan="3" style="border-top: 1px solid black;"></td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">4</td><td style="text-align: center;">6</td></tr> </table> </div> </div>	h	t	u	800	30	6	200	50	4				500	80	2	h	t	u	7	2	8	5	8	2				1	4	6
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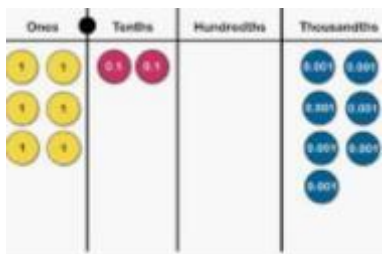
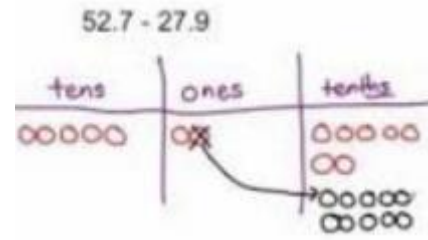
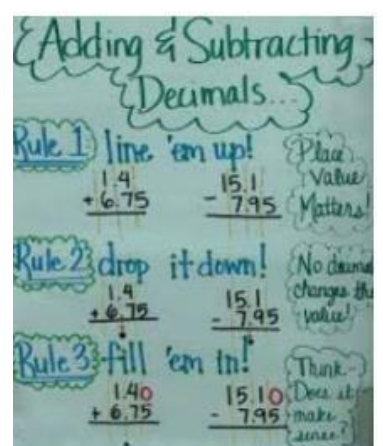
Subtraction- Year 3

Vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Objective and Strategy	Concrete	Pictorial	Abstract
<p style="color: #0070c0; font-weight: bold;">Subtract numbers with up to three-digits, using formal written methods of columnar subtraction</p> <p style="color: #0070c0; font-weight: bold;">Column subtraction (with exchanging)</p> <p style="color: #0070c0; font-weight: bold;">(continued)</p>	<p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p> <p>Make the larger number with the place value counters</p>  <div style="display: flex; align-items: center; margin-left: 10px;"> <div style="border-left: 1px solid black; border-right: 1px solid black; border-bottom: 1px solid black; padding: 5px; margin-right: 5px;">Calculations</div> <div style="border-bottom: 1px solid black; padding: 5px;"> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$ </div> </div> <p>Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.</p> <p>Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.</p>  <div style="display: flex; align-items: center; margin-left: 10px;"> <div style="border-left: 1px solid black; border-right: 1px solid black; border-bottom: 1px solid black; padding: 5px; margin-right: 5px;">Calculations</div> <div style="border-bottom: 1px solid black; padding: 5px;"> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$ </div> </div> <p>Now I can take away eight tens and complete my subtraction</p>  <div style="display: flex; align-items: center; margin-left: 10px;"> <div style="border-left: 1px solid black; border-right: 1px solid black; border-bottom: 1px solid black; padding: 5px; margin-right: 5px;">Calculations</div> <div style="border-bottom: 1px solid black; padding: 5px;"> $\begin{array}{r} \cancel{2}3\cancel{4} \\ - 88 \\ \hline 146 \end{array}$ </div> </div> <p>Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.</p>		

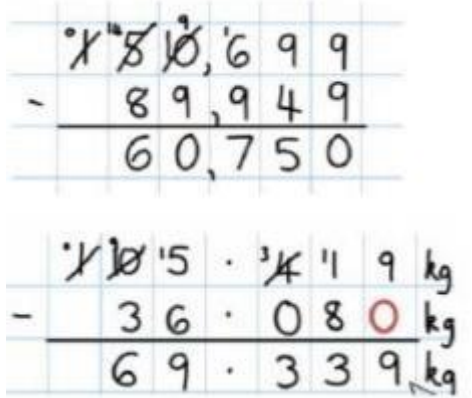
Subtraction- Year 4

Vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Objective and Strategy	Concrete	Pictorial	Abstract								
<p style="color: #0070c0; font-weight: bold;">Subtract numbers with up to 4 digits using the formal written methods appropriate of columnar subtraction where appropriate.</p> <p style="color: #0070c0; font-weight: bold;">Year 4 subtraction with up to 4 digits.</p>	<p>Model process of exchange using Numicon, base ten and then move to place value counters.</p> <p>Use the phrase 'take and make' for exchange—see Y3.</p> <p>The calculation will be shown alongside the model chosen to see the connection.</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr> <th style="padding: 2px 5px;">Model</th> <th style="padding: 2px 5px;">Calculation</th> </tr> <tr> <td style="height: 20px;"></td> <td style="height: 20px;"></td> </tr> </table>	Model	Calculation			<p>Children to draw place value counters and show their exchange—see Y3.</p> <p>The calculation will be shown alongside the model chosen to see the connection.</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr> <th style="padding: 2px 5px;">Model</th> <th style="padding: 2px 5px;">Calculation</th> </tr> <tr> <td style="height: 20px;"></td> <td style="height: 20px;"></td> </tr> </table>	Model	Calculation			<div style="text-align: center;">  </div> <p style="text-align: center;">This will lead to an understanding of subtracting any number including decimals.</p>
Model	Calculation										
Model	Calculation										
<p style="color: #0070c0; font-weight: bold;">Introduce decimal subtraction through context of money.</p>	<p>Children to be encouraged to use counters to represent numbers and take counters away to subtract.</p> 	<div style="text-align: center;">  </div> <p style="text-align: center;">When confident, children can find their own way to record the exchange/regrouping.</p>	<div style="text-align: center;">  </div>								

Subtraction- Year 5 and 6

Vocabulary: equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Subtract with at least 4 digits, including money and measures.</p> <p>Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).</p>	<p>See Year 4</p>	<p>See Year 4</p>	 <p> $\begin{array}{r} \cancel{8}^1 \cancel{1}^0, \cancel{6}^9 \cancel{9}^9 \\ - 89,949 \\ \hline 60,750 \end{array}$ $\begin{array}{r} \cancel{5}^1 \cancel{4}^1 5 \cdot \cancel{3}^4 \cancel{1}^1 9 \text{ kg} \\ - 36 \cdot 080 \text{ kg} \\ \hline 69 \cdot 339 \text{ kg} \end{array}$ </p>

Multiplication: Mental Calculations

These are a selection of mental calculation strategies:

Doubling and halving

Applying the knowledge of doubles and halves to known facts.

e.g. 8×4 is double 4×4

Using multiplication facts

Tables should be taught every day from Year 1 onwards, either as part of the mental oral starter, games or other times as appropriate within the day.

Year 1: 2, 5 and 10 times table

Year 2: 2, 3, 5 and 10 times table

Year 3: 2, 3, 4, 5, 6 and 10 times table

Year 4 Derive and recall all multiplication facts up to 12×12

Years 5 & 6 Derive and recall quickly all multiplication facts up to 12×12

Using and applying multiplication facts

Children should be able to utilise their tables knowledge to derive other facts.

E.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\,000$, $0.3 \times 7 = 2.1$ etc.

Use closely related facts already known

$$13 \times 11 = (13 \times 10) + (13 \times 1)$$

$$= 130 + 13$$

$$= 143$$

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Partitioning


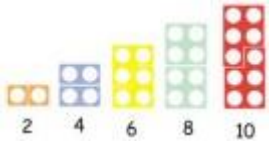
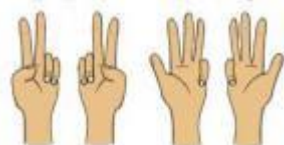

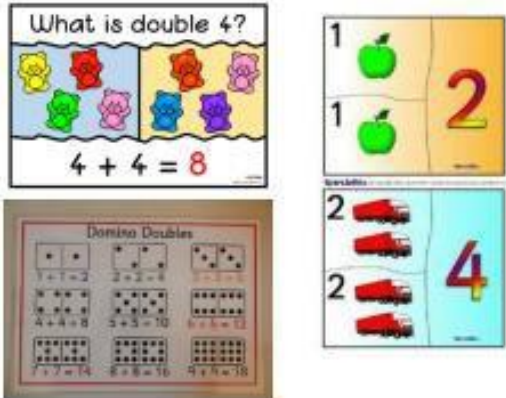
$$\begin{aligned}23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= 80 + 12 \\ &= 92\end{aligned}$$

Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

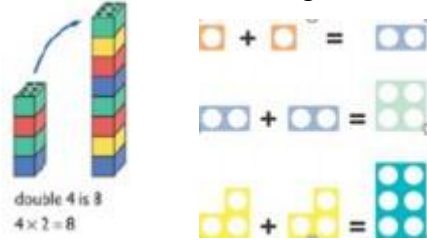

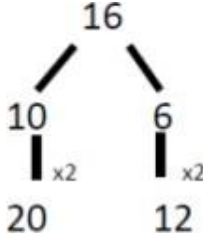

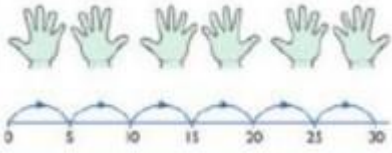

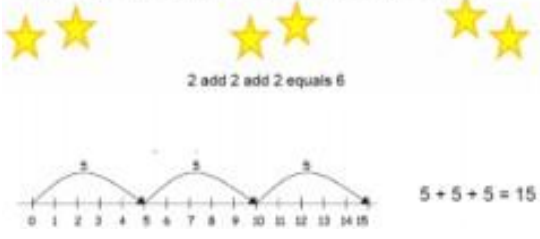

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

Multiplication: Written Calculations

Multiplication- EYFS															
Objectives	Concrete	Pictorial	Abstract												
<p>- Solve problems including doubling.</p>	 <p>Counting and other Maths resources for children to make 2 equal groups.</p>  <p>2 4 6 8 10</p>  <p>Physical and real life examples that encourage children to see concept of doubling as adding two equal groups.</p>  <p>Double 1 eyes socks</p>	 <p>What is double 4? $4 + 4 = 8$</p> <p>Domino Doubles</p> <p>1 + 1 = 2 2 + 2 = 4 3 + 3 = 6 4 + 4 = 8 5 + 5 = 10 6 + 6 = 12 7 + 7 = 14 8 + 8 = 16 9 + 9 = 18</p> <p>1 2 2 4</p> <p>Pictures and icons that encourage children to see concept of doubling as adding two equal groups.</p>	<table border="1"> <tbody> <tr> <td>$1+1=$</td> <td>$7+7=$</td> </tr> <tr> <td>$2+2=$</td> <td>$8+8=$</td> </tr> <tr> <td>$3+3=$</td> <td>$9+9=$</td> </tr> <tr> <td>$4+4=$</td> <td>$10+10=$</td> </tr> <tr> <td>$5+5=$</td> <td>$11+11=$</td> </tr> <tr> <td>$6+6=$</td> <td>$12+12=$</td> </tr> </tbody> </table> <p>Adding calculations to model adding two equal groups.</p>	$1+1=$	$7+7=$	$2+2=$	$8+8=$	$3+3=$	$9+9=$	$4+4=$	$10+10=$	$5+5=$	$11+11=$	$6+6=$	$12+12=$
$1+1=$	$7+7=$														
$2+2=$	$8+8=$														
$3+3=$	$9+9=$														
$4+4=$	$10+10=$														
$5+5=$	$11+11=$														
$6+6=$	$12+12=$														

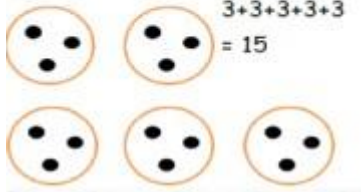

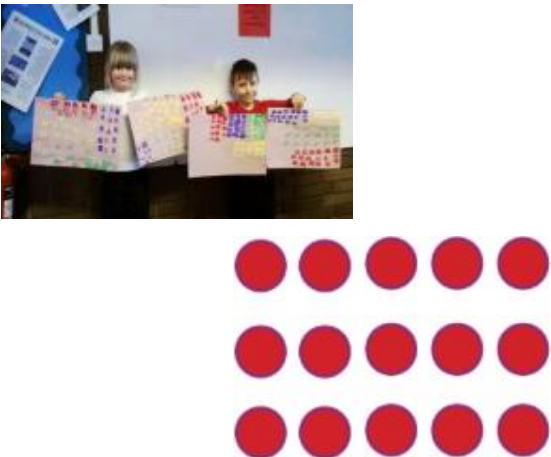
Multiplication- Year 1

Vocabulary: groups of, lots of, times, array, altogether, multiply

Objective and Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers.</p> <p style="text-align: center;">Double 4 is 8</p> 	 <p style="text-align: right;">Partition each number and then double each part before recombining it back together.</p>
Counting in multiples	 <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p style="text-align: center;">2, 4, 6, 8, 10</p> <p style="text-align: center;">5, 10, 15, 20, 25, 30</p>
Repeated addition	<p>Use different objects to add equal groups.</p> 	<p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>  <p style="text-align: center;">$2 \text{ add } 2 \text{ add } 2 \text{ equals } 6$</p> <p style="text-align: right;">$5 + 5 + 5 = 15$</p>	<p>Write addition sentences to describe objects and pictures.</p>  <p style="text-align: center;">$2 + 2 + 2 + 2 + 2 = 10$</p>

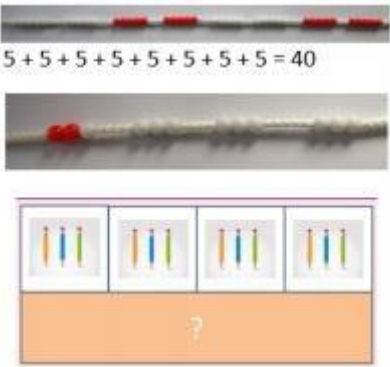
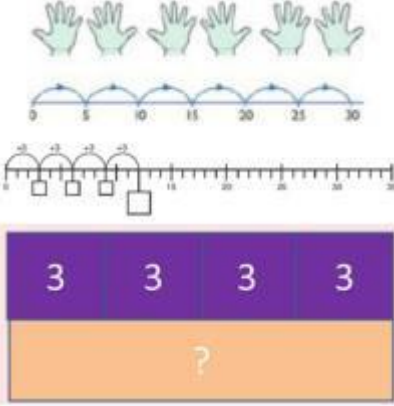

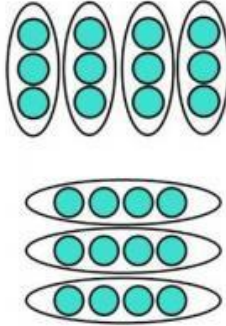

Multiplication- Year 1

Vocabulary: groups of, lots of, times, array, altogether, multiply

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Repeated addition (continued)</p>		<p>Use pictorial including number lines to solve problems.</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> 	
<p>Understanding arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2 etc.</p> 		<p>$3 \times 2 = 6$</p> <p>$2 \times 5 = 10$</p>


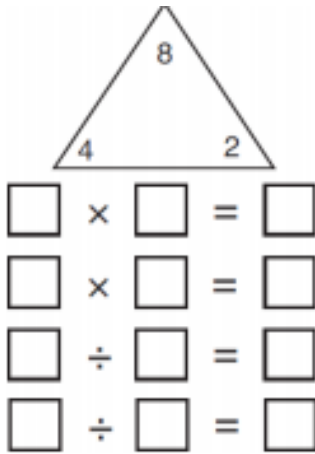
Multiplication- Year 2

Vocabulary: Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)</p>	<p>Count the groups as children are skip counting. Children may use their fingers as they are skip counting. Use bar models.</p>  <p>$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$</p>	<p>Number lines, counting sticks and bar models should be use to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30</p> <p style="text-align: center;">$4 \times 3 = \square$</p>
<p>Multiplication is commutative</p>	<p>Create arrays using counters, cubes and Numicon.</p>  <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p>	<p>Use representations of arrays to show different calculations and explore commutatively.</p> 	<p>$12 = 3 \times 4$</p> <p>$12 = 4 \times 3$</p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$</p>

Multiplication- Year 2

Vocabulary: Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.

Objective and Strategy	Concrete	Pictorial	Abstract
<p style="color: #0070c0; font-weight: bold;">Using the Inverse</p> <p style="color: #0070c0; font-weight: bold;">This should be taught alongside division, so pupils learn how they work alongside each other.</p>			<p>Show all 8 related fact family sentences:</p> <p>$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$</p>

Multiplication- Year 3

Vocabulary: Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up

Objective and Strategy

Concrete

Pictorial

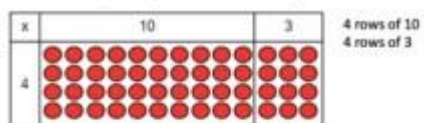
Abstract

Multiplying two-digit number by a one-digit number

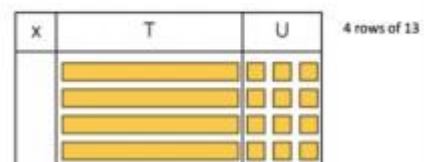
Grid method progressing to the formal method

Solving problems including missing number problems, integer scaling problems

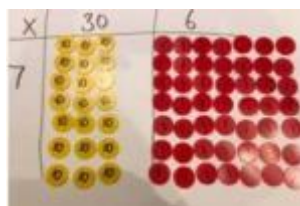
Show the link with arrays to first introduce the grid method.



Move on to using Base 10 to move towards a more compact method.



Move on to place value counters to show how we are finding groups of a number.



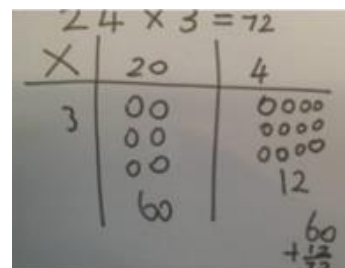
Add up each column, starting with the ones making any exchanges needed.

The calculation will be shown alongside the model to see the connection.

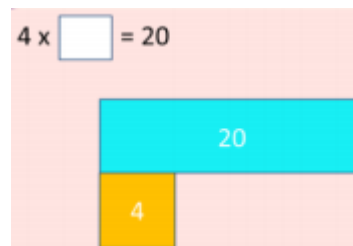
Model	Calculation

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below:



Bar model are used to explore missing numbers.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

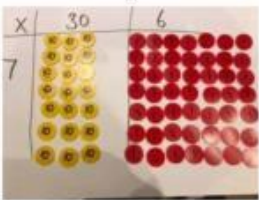

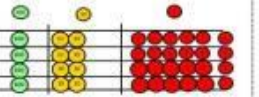
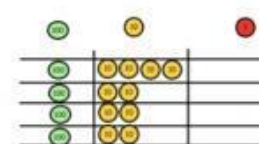

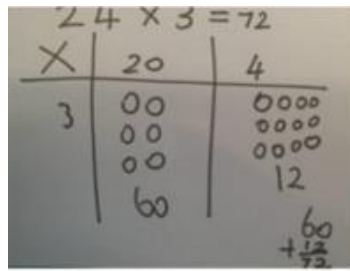
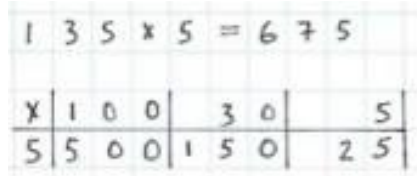
TO X O



Children to add up each column to find the answer.

Multiplication- Year 4

Vocabulary: Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Multiply two-digit and three-digit numbers by a one-digit number using formal written layout</p> <p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Multiplying numbers by 1 digit (year 4 expectation)</p>	 <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</p>  <p>Calculations 4×126</p> <p>Fill each row with 126.</p>  <p>Calculations 4×126</p> <p>Add up each column, starting with the ones making any exchanges needed.</p>  <p>Then you have your answer.</p> 	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below:</p> 	<p>HTO X O</p>  <p>Children to add up each column to find the answer.</p>

Multiplication- Year 4

Vocabulary: Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive

Objective and Strategy

Concrete

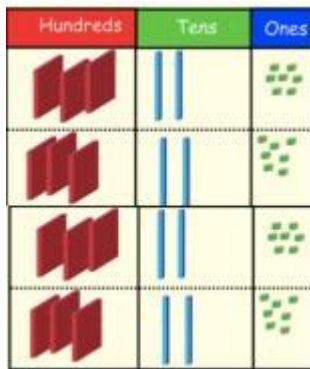
Pictorial

Abstract

Column Multiplication

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping.

E.g. $321 \times 2 = 642$



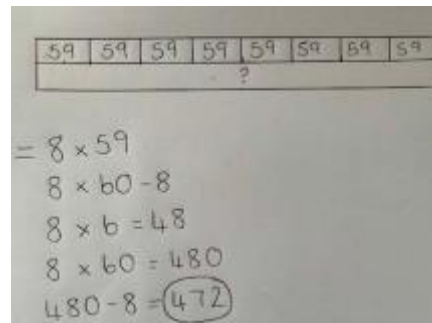
It is important at this stage that they always multiply the ones first.

The corresponding long multiplication is modelled alongside:

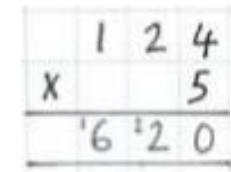
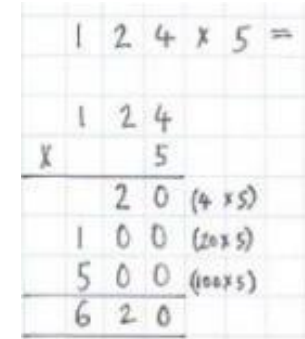
Model	Calculation

The grid method may be used to show how this relates to a formal written method:

x	100	20	4
5	500	100	20




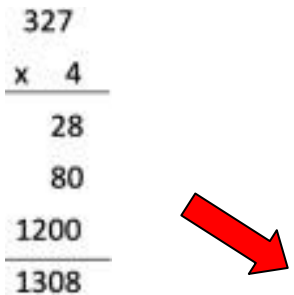
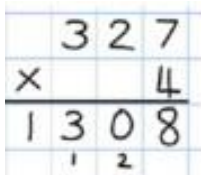
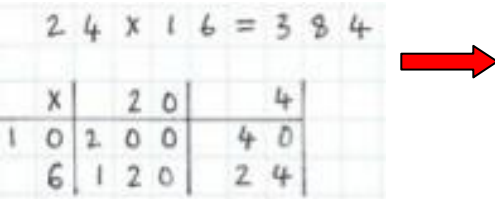
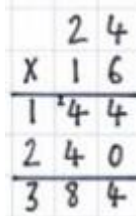
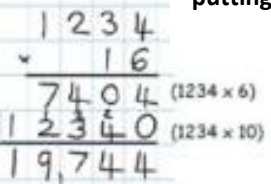
Bar modelling and number line can support learners when solving problems with multiplication alongside the formal written methods.



This may lead to a compact method.

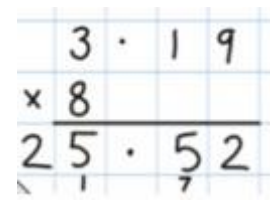
Multiplication- Year 5

Vocabulary: Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed

Objective and Strategy	Concrete	Pictorial	Abstract				
<p>Multiply numbers up to 4-digits by a one-digit number using the format written method, including long multiplication for 2-digit numbers</p> <p>Column multiplication for 3 and 4 digits x 1 digit</p>	<p>See Year 4</p>		 <p style="text-align: center;">This may lead to a compact method.</p> 				
<p>Column multiplication (long multiplication)</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Model</th> <th style="padding: 5px;">Calculation</th> </tr> </thead> <tbody> <tr> <td style="height: 30px;"></td> <td></td> </tr> </tbody> </table>	Model	Calculation			<p>Moving forward, multiply by a 2-digit number showing the different rows within the grid method.</p> 	 <p style="margin-left: 20px;">24 x 6 on the first row. (6 x 4 = 24, carrying the 2 for the 20, then 6 x 2)</p> <p style="margin-left: 20px;">24 x 10 on the second row. Show multiplying by 10 by putting zero in the units first.</p> 
Model	Calculation						

Multiplication- Year 6

Vocabulary: Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Multiply multi-digit numbers up to 4 digits by a 2-digit number using the formal written method of long multiplication.</p>	<p>See Year 5</p>	<p>See Year 5</p>	<p>See Year 5</p>
<p>Multiply decimal up to 2 decimal place by a single digit</p>			<p>Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer.</p> 

Division: Mental Calculations

These are a selection of mental calculation strategies:

Doubling and halving

Knowing that halving is dividing by 2.

Deriving and recalling division facts

Tables should be taught every day from Year 1 onwards, either as part of the mental oral starter, games or other times as appropriate within the day.

Year 1: 2, 5 and 10 times table

Year 2: 2, 3, 5 and 10 times table

Year 3: 2, 3, 4, 5, 6 and 10 times table

Year 4 Derive and recall all multiplication facts up to 12 x 12

Years 5 & 6 Derive and recall quickly all multiplication facts up to 12 x 12

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc.

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

$378 \div 21$ $378 \div 3 = 126$ $378 \div 21 = 18$

$126 \div 7 = 18$




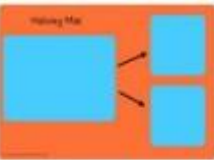


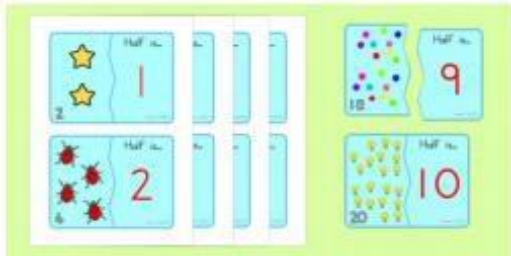

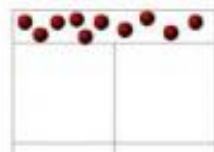
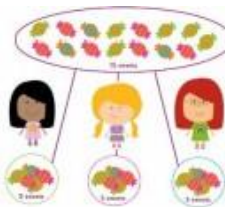
Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div 1.1$?

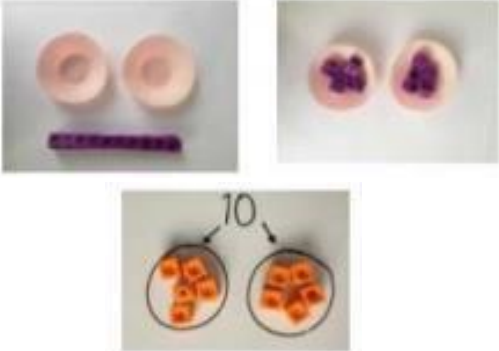
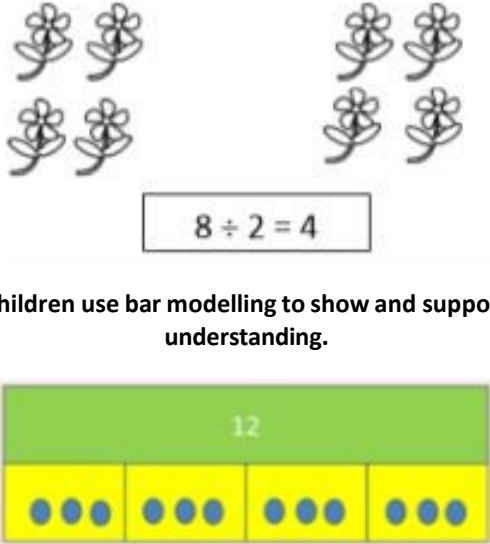
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS

Division: Written Calculations

Division- EYFS			
Objectives	Concrete	Pictorial	Abstract
<p>-Solve problems including halving and sharing.</p> <ul style="list-style-type: none"> • Halving a whole, halving a quantity of objects. • Sharing a quantity of objects. 	  <p>Children have the opportunity to physically cut objects, food or shapes in half.</p>  <p>Counting and other maths resources for children to share into two equal groups.</p>   <p>Use visual supports such as halving mats and part whole, with the physical objects and resources that can be manipulated.</p>  <p>Counting and other maths resources for children to explore sharing between 3 or more.</p>	 <p>Pictures and icons that encourage children to see concept of halving in relation to addition and subtraction knowledge. i.e. knowing that 4 is made up of 2 groups of 2, so half of 4 is 2.</p>   <p>Bar model with pictures or icons to support understanding of finding 2 equal parts of a number, to further understand how two halves make a whole.</p>  <p>Pictures for children to create and visualise 3 or more equal groups.</p>	

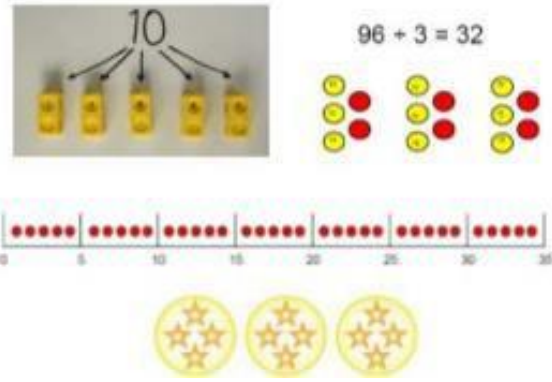
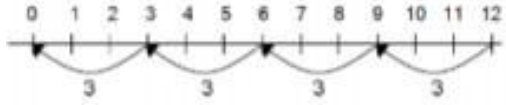
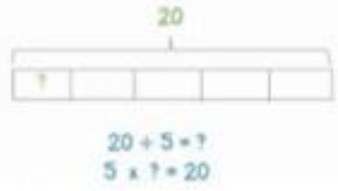
Division- Year 1

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing (sharing objects into groups)</p>	 <p>I have 10 cubes; can you share them equally into 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>Children use bar modelling to show and support understanding.</p> <p>$12 \div 4 = 3$</p>	<p>Share 9 buns between three people.</p> <p>$9 \div 3 = 3$</p>



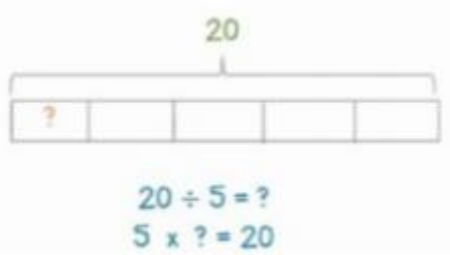

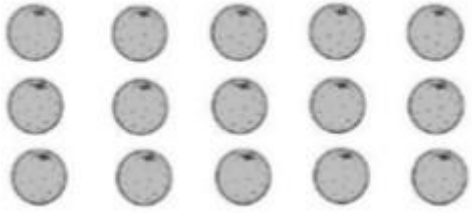
Division- Year 2

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p> 	<p>Use a number line to show jumps in groups. The number of jumps equal the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p> 	<p style="text-align: center;">$28 \div 7 = 4$</p> <p style="text-align: center;">Divide 28 into 7 groups. How many are in each group?</p>

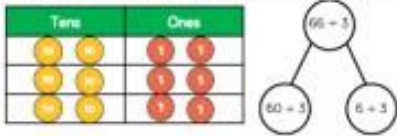
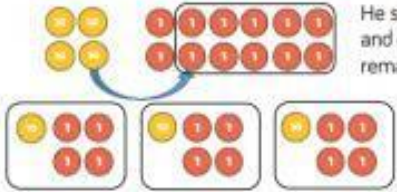
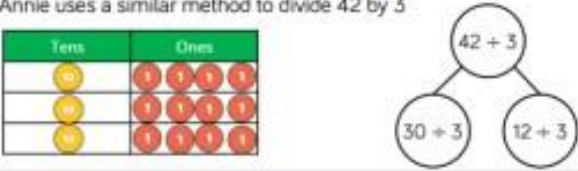
Division- Year 3

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> <p>$96 \div 3 = 32$</p> 	<p>Continue to use bar modelling to aid solving division problems.</p> 	<p>How many groups of 6 in 24?</p> <p>$24 \div 6 = 4$</p>
<p>Division with arrays</p>	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>E.g. $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences.</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> <p>$7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$</p>

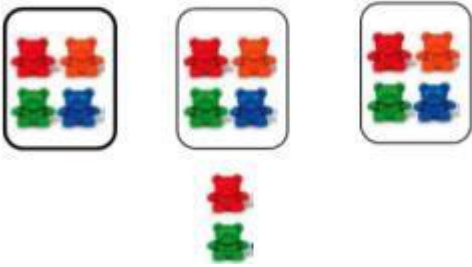
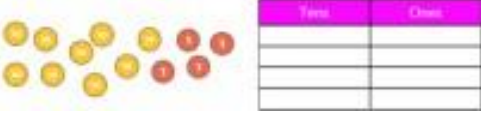


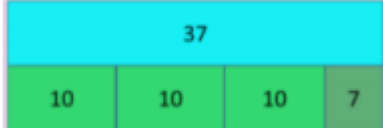

Division- Year 3

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Divide 2-digit numbers by a 1-digit number by partitioning into tens and ones using a place value grid</p>	<p>Eva uses a place value grid and part-whole model to solve $66 \div 3$</p> 	<p>See part-whole model</p>	
<p>Divide numbers that involve exchanging between the tens and ones. The answers do not have remainders</p>	<p>Ron uses place value counters to divide 42 into three equal groups.</p>  <p>He shares the tens first and exchanges the remaining ten for ones.</p> <p>Then he shares the ones. $42 \div 3 = 14$</p>	<p>Annie uses a similar method to divide 42 by 3</p>  <p>Children may use pictorial representation for the place value counters, alongside the part-whole model.</p> <p>Children use their times-tables to partition the number into multiples of the divisor.</p>	<p>$96 \div 8$ $96 \div 4$ $96 \div 3$ $96 \div 6$</p> <p>Compare the statements using $<$, $>$ or $=$</p> <p>$48 \div 4$ <input type="radio"/> $36 \div 3$</p> <p>$52 \div 4$ <input type="radio"/> $42 \div 3$</p> <p>$60 \div 3$ <input type="radio"/> $60 \div 4$</p>


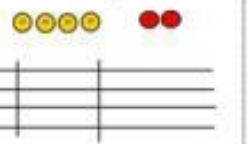

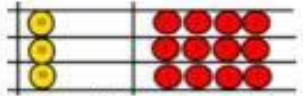
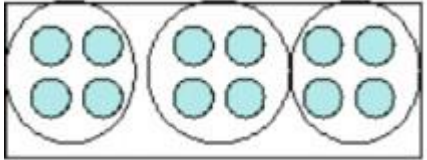
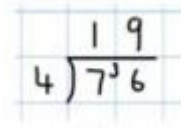
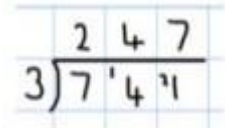
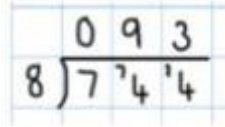
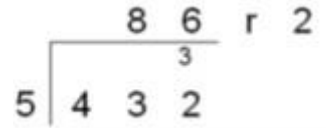
Division- Year 3

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Division with a remainder</p>	<p style="text-align: center;">$14 \div 3 =$</p> <p>Divide objects between groups and see how much is left over</p>  <p>Moving on to:</p> <p>Use place value counters to work out $94 \div 4$ Did you need to exchange any tens for ones? Is there a remainder?</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly shows a remainder.</p>  <p>Use bar models to show division with remainders.</p> 	<p>Complete written division and show the remainder using r.</p> 

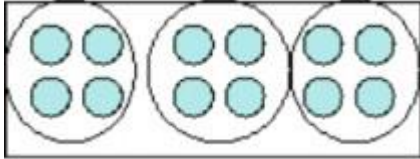
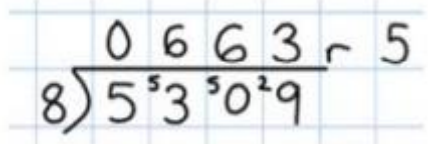
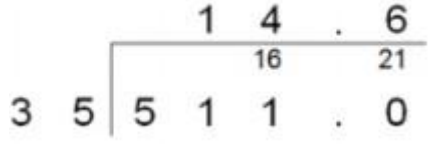
Division- Year 4

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive

Objective and Strategy	Concrete	Pictorial	Abstract				
<p>Divide up to 3-digit numbers by 1-digit</p> <p>Short Division</p>	<p>$96 \div 3$</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">Tens</td> <td style="text-align: center;">Units</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">2</td> </tr> </table>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	Tens	Units	3	2	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p>   <p>Children should be aware that a 0 is used to keep place value, if the number is not divisible.</p>  <p>Move onto divisions with a remainder.</p> 
Tens	Units						
3	2						

Division- Year 5

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method

Objective and Strategy	Concrete	Pictorial	Abstract
<p>Divide at least 4-digit numbers by 1-digit.</p> <p>Interpret remainders appropriately for the context.</p> <p>Short Division</p>	<p>See Year 4</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	 <p>Finally move into decimal places to divide the total accurately.</p> 

Division- Year 6

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method

Objective and Strategy

Abstract

Long Division

Step 1- A remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041\text{R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400\text{R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens)

8 goes into 7 zero times, and leaves a remainder of 7.

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Division- Year 6

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method

Objective and Strategy

Abstract

Long Division

Step 1- A remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \\ \hline 2 \overline{)58} \end{array}$ <p>Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \\ \hline 2 \overline{)58} \\ -4 \\ \hline 1 \end{array}$ <p>To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ \hline 2 \overline{)58} \\ -4 \downarrow \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>
$\begin{array}{r} \text{t o} \\ 29 \\ \hline 2 \overline{)58} \\ -4 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ \hline 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ \hline 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

Division- Year 6

Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method

Objective and Strategy

Abstract

Long Division

Step 1- A remainder in any of the place values

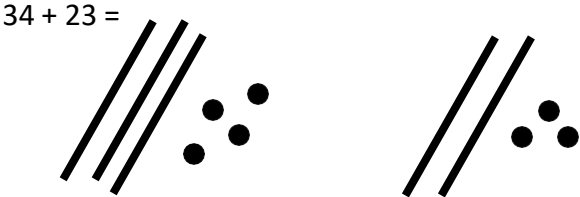
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds + 2 = 1 hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \\ -2 \\ \hline 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 138 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>

Representing Manipulatives

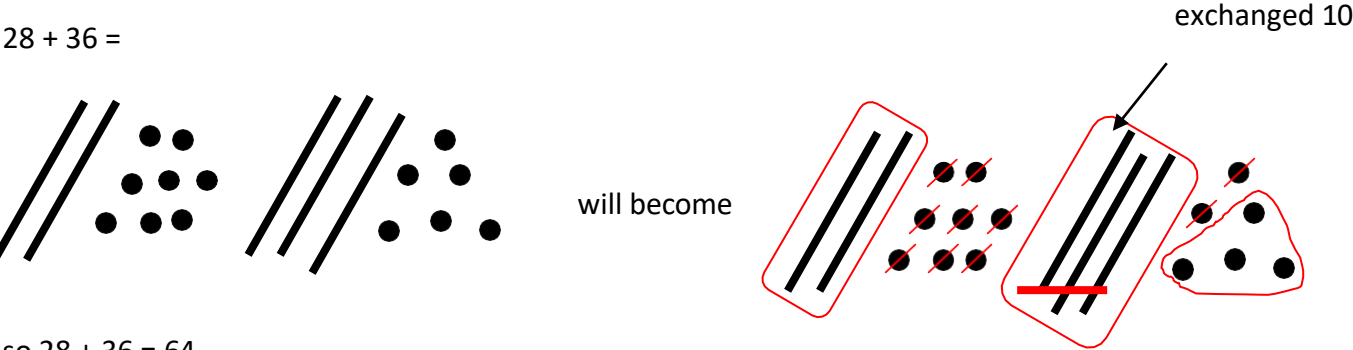
Dienes/Base 10

Children can record calculations using their own drawings of Base 10 equipment (as slanted lines for the tens rods and dots for the ones blocks).

Year 2 Examples:

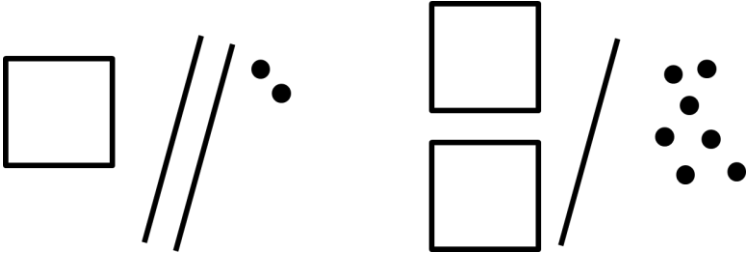


With exchange:



It is important that children circle the remaining tens and units/ones after exchange to identify the amount remaining.

This method can also be used with adding three digit numbers, e.g. 122 + 217 using a square as the



representation of 100 (a cube would be used for a representation of 1000).

Place Value Counters

Children can record calculations using their drawings of place value counters as coloured dots (see below):

Ones = red
 Tens = yellow
 Hundreds = green
 Thousands = blue



Examples:

Put $<$, $>$ or $=$ in the circles to make the statement correct.

100s	10s	1s
●	●● ●	●● ●●

○

100s	10s	1s
●● ●	●● ●●	●

○

100s	10s	1s
●● ●●	●	●● ●●

Write the number represented in the place value grid

Hundreds	Tens	Ones
● ●	●● ●● ●● ●	● ●

Resources required to support the CPA approach (depending on the year group) for this calculation policy:

- 1. Ten frames**
- 2. Straws/pipe cleaners**
- 3. Bead strings (to 20 and 100)**
- 4. Base 10/Dienes**
- 5. Place value grids**
- 6. Number lines**
- 7. Two-coloured counters**
- 8. Part-whole templates**
- 9. Place value counters (KS2)**
- 10. Multi-link cubes**